Kuwait University	Math 101	Date:	October 28, 2004
Dept. of Math. & Comp. Sci.	First Exam	Duration:	75 minutes

Calculators, mobile phones, pagers and all other mobile communication equipment are not allowed

Answer the following questions:

1. Use the definition of the limit to show that:

$$\lim_{x\to 4}(7-2x)=-1.$$ (3 pts.)

2. Evaluate the following limits, if they exist:

(a) $\lim_{x\to 1}\left[(x-1)^2\sin\left(\frac{\pi}{x-1}\right)+2\right]$ (3 pts.)

(b) $\lim_{x\to 0}\frac{2x+\sin 3x}{x-\tan 6x}$ (3 pts.)

3. Find the x–coordinates of the points at which the function f is discontinuous, where

$$f(x)=\frac{x^3-8}{x^2-5x+6}.$$

Classify the types of discontinuity of f as removable, jump, or infinite. (4 pts.)

4. (a) State The Intermediate Value Theorem. (1 pt.)

(b) Show that $f(x)=3x^4+6x^3+x^2+x+1$, has a real root. (2 pts.)

5. Use the definition of the derivative to find $f'(3)$, where $f(x)=\sqrt{x+1}$. (3 pts.)

6. Find the points on the graph of $f(x)=\dfrac{x^2}{x+1}$, at which the tangent line is horizontal.

(3 pts.)

7. Find $f'(x)$, where $f(x)=\sin^4(5x^3+x+2)-\csc\sqrt{x^2+1}$. (3 pts.)

1. Let $\varepsilon > 0$ such that: $|(7-2x)-(-1)| < \varepsilon \iff |x-4| < \frac{\varepsilon}{2}$. Take $\boxed{0 < \delta \le \frac{\varepsilon}{2}}$. So, for every $\varepsilon > 0$ there is a $\delta > 0$ such that if $0 < |x-4| < \delta$, then $|(7-2x)-(-1)| < \varepsilon$.

2. (a) $-1 \le \sin\left(\frac{\pi}{x-1}\right) \le 1$ for $x \ne 1$. $\Longrightarrow$ $-(x-1)^2 \le (x-1)^2 \sin\left(\frac{\pi}{x-1}\right) \le (x-1)^2$. $\lim\limits_{x\to 1} -(x-1)^2 = 0 = \lim\limits_{x\to 1}(x-1)^2$, from the Squeeze Theorem:

$$\lim_{x\to 1}(x-1)^2 \sin\left(\frac{\pi}{x-1}\right) = 0 \Longrightarrow \lim_{x\to 1}\left[(x-1)^2\sin\left(\frac{\pi}{x-1}\right)+2\right] = 0+2 = \boxed{2}.$$

(b) $$\lim_{x\to 0}\frac{2x+\sin 3x}{x-\tan 6x} = \lim_{x\to 0}\frac{\not{x}\left(2+\frac{\sin 3x}{x}\right)}{\not{x}\left(1-\frac{\tan 6x}{x}\right)} = \frac{2+3\lim\limits_{3x\to 0}\frac{\sin 3x}{3x}}{1-6\lim\limits_{6x\to 0}\frac{\tan 6x}{6x}} = \frac{2+3(1)}{1-6(1)} = \boxed{-1}.$$

3. $x^2-5x+6 = (x-2)(x-3)$.

$f(2)$ is undefined, $\lim\limits_{x\to 2} f(x) = \lim\limits_{x\to 2}\frac{(x-2)(x^2+2x+4)}{(x-2)(x-3)} = \boxed{-12} \Longrightarrow$ The graph of f has a *removable* discontinuity at $\boxed{x=2}$.

$f(3)$ is undefined, $\lim\limits_{x\to 3^\pm} f(x) = \boxed{\pm\infty} \Longrightarrow$ The graph of f has an *infinite* discontinuity at $\boxed{x=3}$.

4. (a) $f(0) = 1 > 0$, $f(-1) = -2 < 0$ & f is continuous on $[-1,0]$ [polynomial function] $\Longrightarrow$ From The Intermediate Value Theorem, there is a $c \in (-1,0)$ such that $f(c) = 0 \Longrightarrow c$ is a real root for f.

5. $\frac{f(x)-f(3)}{x-3} = \frac{\sqrt{x+1}-2}{x-3} \times \frac{\sqrt{x+1}+2}{\sqrt{x+1}+2} = \frac{(x+1)-4}{(x-3)\left(\sqrt{x+1}+2\right)} = \frac{1}{\sqrt{x+1}+2}$, for $x \ne 3$. $\Longrightarrow f'(3) = \lim\limits_{x\to 3}\frac{f(x)-f(3)}{x-3} = \boxed{\frac{1}{4}}$.

6. $f'(x) = \frac{x^2+2x}{(x+1)^2} = \frac{x(x+2)}{(x+1)^2}$. For horizontal tangent $f'(x) = 0 \Longrightarrow$ $x = 0$ or $x = -2$. $\Longrightarrow$ The graph of f has horizontal tangent lines at $P_1(0,0)$ and $P_2(-2,-4)$. $[f(0) = 0, f(-2) = -4]$.

7. $f'(x) = 4(15x^2+1)\cos(5x^3+x+2)\sin^3(5x^3+x+2) + \frac{x}{\sqrt{x^2+1}}\csc\sqrt{x^2+1}\cot\sqrt{x^2+1}$.